1. Philip and James are racing car drivers. Philip's lap times, in seconds, are normally distributed with mean 90 and variance 9. James' lap times, in seconds, are normally distributed with mean 91 and variance 12. The lap times of Philip and James are independent. Before a race, they each take a qualifying lap.

    (a) Find the probability that James' time for the qualifying lap is less than Philip's.

    (4)

    The race is made up of 60 laps. Assuming that they both start from the same starting line and lap times are independent,

    (b) find the probability that Philip beats James in the race by more than 2 minutes.

    (5)
    (Total 9 marks)

2. The random variable A is defined as

    $$A = 4X - 3Y$$

    where $X \sim N(30, 3^2)$, $Y \sim N(20, 2^2)$ and X and Y are independent.

    Find

    (a) E(A),

    (2)

    (b) Var(A).

    (3)

    The random variables Y_1, Y_2, Y_3 and Y_4 are independent and each has the same distribution as Y. The random variable B is defined as

    $$B = \sum_{i=1}^{4} Y_i$$

    (c) Find P($B > A$).

    (6)
    (Total 11 marks)

3. A set of scaffolding poles come in two sizes, long and short. The length L of a long pole has the normal distribution $N(19.7, 0.5^2)$. The length S of a short pole has the normal distribution $N(4.9, 0.2^2)$. The random variables L and S are independent.

A long pole and a short pole are selected at random.

(a) Find the probability that the length of the long pole is more than 4 times the length of the short pole.

(7)

Four short poles are selected at random and placed end to end in a row. The random variable T represents the length of the row.

(b) Find the distribution of T.

(3)

(c) Find $P(|L - T| < 0.1)$.

(5)
(Total 15 marks)

4. The workers in a large office block use a lift that can carry a maximum load of 1090 kg. The weights of the male workers are normally distributed with mean 78.5 kg and standard deviation 12.6 kg. The weights of the female workers are normally distributed with mean 62.0 kg and standard deviation 9.8 kg.
Random samples of 7 males and 8 females enter the lift.

(a) Find the mean and variance of the total weight of the 15 people that enter the lift.

(4)

(b) Comment on any relationship you have assumed in part (a) between the two samples.

(1)

(c) Find the probability that the maximum load of the lift will be exceeded by the total weight of the 15 people.

(4)
(Total 9 marks)

5.   A workshop makes two types of electrical resistor.

The resistance, *X* ohms, of resistors of Type *A* is such that $X \sim N(20, 4)$.

The resistance, *Y* ohms, of resistors of Type *B* is such that $Y \sim N(10, 0.84)$.

When a resistor of each type is connected into a circuit, the resistance *R* ohms of the circuit is given by $R = X + Y$ where *X* and *Y* are independent.

Find

(a)   E(*R*),

(1)

(b)   Var(*R*),

(2)

(c) $P(28.9 < R < 32.64)$.

(6)
(Total 9 marks)

6.   A manufacturer produces two flavours of soft drink, cola and lemonade. The weights, C and L, in grams, of randomly selected cola and lemonade cans are such that $C \sim N(350, 8)$ and $L \sim N(345, 17)$.

(a)   Find the probability that the weights of two randomly selected cans of cola will differ by more than 6 g.

(6)

One can of each flavour is selected at random.

(b)   Find the probability that the can of cola weighs more than the can of lemonade.

(6)

Cans are delivered to shops in boxes of 24 cans. The weights of empty boxes are normally distributed with mean 100 g and standard deviation 2 g.

(c)   Find the probability that a full box of cola cans weighs between 8.51 kg and 8.52 kg.

(6)

(d)   State an assumption you made in your calculation in part (c).

(1)
(Total 19 marks)

7.   The random variable D is defined as

$$D = A - 3B + 4C$$

where $A \sim N(5, 2^2)$, $B \sim N(7, 3^2)$ and $C \sim N(9, 4^2)$, and A, B and C are independent.

(a)   Find $P(D < 44)$.

(9)

The random variables B_1, B_2 and B_3 are independent and each has the same distribution as B.
The random variable X is defined as

$$X = A - \sum_{i=1}^{3} B_i + 4C.$$

(b)   Find $P(X > 0)$.

(7)
(Total 16 marks)

8.   Given the random variables $X \sim N(20, 5)$ and $Y \sim N(10, 4)$ where X and Y are independent, find

(a) $E(X - Y)$,

(2)

(b) $Var(X - Y)$,

(2)

(c) $P(13 < X - Y < 16)$.

(5)
(Total 9 marks)

1. (a) $[P \sim N(90,9) \text{ and } J \sim N(91,12)]$

$(J - P) \sim N(1, 21)$

$\begin{aligned} P(J < P) &= P(J - P < 0) & \text{M1, A1} \\ &= P\left(Z < \frac{0-1}{\sqrt{21}} \right) & \text{dM1} \\ &= P(Z < -0.2182...) \\ &= 1 - 0.5871 = 0.4129 & \text{awrt (\textbf{0.413 $\sim$ 0.414}) A1} \quad 4 \end{aligned}$

calculator (0.4136….)

Note

1st M1   for attempting $J - P$ and $E(J - P)$ or $P - J$ and $E(P - J)$

1st A1   for variance of 21 (Accept $9 + 12$). Ignore any slip in μ here.

2nd dM1   for attempting the correct probability and standardising with
their mean and sd.
This mark is dependent on previous M so if $J - P$ (or $P - J$) is
not being used score M0
If their method is not crystal clear then they must be attempting
$P(Z < $ -ve value$)$ or $P(Z > $ +ve value$)$ i.e. their probability <u>after</u>
standardisation should lead to a prob. < 0.5 so e.g. $P(J - P < 0)$
leading to 0.5871 is M0A0 unless the M1 is clearly earned.

2nd A1   for awrt 0.413 or 0.414

The first 3 marks may be implied by a correct answer

(b) $X = (J_1 + J_2 + ... + J_{60}) - (P_1 + P_2 + ... + P_{60})$ M1

$E(X) = 60 \times 91 - 60 \times 90 = 60$ [stated as $E(X) = 60$ or $X \sim N(60, …)$] B1

$\text{Var}(X) = 60 \times 9 + 60 \times 12 = 1260$ A1

$\begin{aligned} P(X > 120) &= P\left(Z > \frac{120 - 60}{\sqrt{1260}} \right) & \text{M1} \\ &= P(Z > 1.69030...) \\ &= 1 - 0.9545 = 0.0455 & \text{awrt (\textbf{0.0455}) A1} \quad 5 \end{aligned}$

Note

1st M1   for a clear attempt to identify a correct form for *X*. This
may be implied by correct variance of 1260

B1   for E(*X*) = 60. Can be awarded even if they are using
$X = 60J – 60P$. Allow P – J and –60

1st A1   for a correct variance. If 1260 is given the M1 is scored
by implication.

2nd M1   for attempting a correct probability and standardising with
120 and their 60 and 1260
If the answer is incorrect a full <u>expression</u> must be seen
following through their values for M1 e.g.

$$P\left(Z > \frac{120 - \text{their } 60}{\sqrt{\text{their variance}}} \right).$$ If using –60, should get

$$P\left(Z < \frac{-120 - -60}{\sqrt{\text{their variance}}} \right)$$

Use of means

Attempt to use $\bar{J} - \bar{P}$ for 1st M1, E($\bar{J} - \bar{P}$) = 1 for B1 and
Var($\bar{J} - \bar{P}$) = 0.35 for A1
Then 2nd M1 for standardisation with 2, and their 1 and 0.35

[9]

2.   (a)   E(4*X* – 3*Y*)=4E(*X*) – 3E(*Y*)   M1

                $= 4\times30 – 3 \times20$

                = 60   A1   2

Note

M1   for correct use of E $(aX + bY)$
formula

     (b)   Var(4*X* – 3*Y*) = 16 Var (*X*) + 9 Var (*Y*)   16 or 9; adding   M1; M1

                $= 16 \times 9 + 9 \times 4$

                = 180   A1   3

Note

1st M1   for 16Var(*X*) or 9Var(*Y*)

2nd M1   for <u>adding</u> variances

               Key points are the 16, 9 and +.
Allow slip e.g using Var(*X*)=4
etc to score Ms

(c) E(B) = 80   B1

Var (B) = 16   B1

E($B - A$) = 20   E(B)-E(A)   M1

Var ($B - A$) = 196   ft on 180 and 16   A1ft

$$P(B - A > 0) = P\left(Z > \frac{-20}{\sqrt{196}}\right)$$

$$= [P(Z > -1.428....)]$$ stand. using their
                                                          mean and var   dM1

= 0.923 …   awrt
                                                          0.923 – 0.924   A1   6

Note

1ˢᵗ M1   for attempting $B - A$ and E($B - A$)
              or $A - B$ and E($A - B$)

              This mark may be implied by an attempt
              at a correct probability

              e.g. $P\left(Z > \dfrac{0 - (80 - 60)}{\sqrt{180 + 16}}\right)$.

              To be implied we must see the "0"

1ˢᵗ A1ft   for Var($B - A$) can ft their Var(A) = 180
              and their Var(B) = 16

2ⁿᵈ dM1   Dependent upon the 1ˢᵗ M1 in part (c).

              for attempting a correct probability i.e.
              P(B–A>0) or P(A–B < 0) and standardising
              with their mean and variance.

              They must standardise properly with the 0
              to score this mark

2ⁿᵈ A1   for awrt 0.923 ~ 0.924

[11]

3. (a)   Lte $X = L - 4S$ then $E(X) = 19.7 - 4 \times 4.9 = 0.1$ M1, A1

Var(X) = Var$(L) + 4^2$Var$(S) = 0.5^2 + 16 \times 0.2^2$ M1, M1

= 0.89   A1

$P(X > 0) = [P(Z > -0.10599...)]$ M1

AWRT (<u>0.542 – 0.544</u>)   A1   7

1st M1   for defining X and attempting $E(X)$

1st A1   for 0.1. Answer only will score both marks.

2nd M1   for Var$(L) + ...$

3rd M1   for... 4^2 Var(S). For those who don't attempt $L - 4S$
this will be their only mark in (a).

2nd A1   for 0.89

4th M1   for attempting a correct probability, a correct expression
and attempt to find, which should involve some
standardisation ; ft their $\sqrt{0.89}$ and their 0.1.
If 0.1 is used for $E(X)$ answer should be > 0.5, otherwise M0.

(b) $T = S_1 + S_2 + S_3 + S_4$ (may be implied by 0.16)   M1

$T - N(19.6, 0.16)$ $E(T) - 19.6$ B1

Var$(T) = 0.16$ or 0.4^2 A1   3

(c)   Let $Y = L - T$ $E(Y) = E(L) - E(T) = [0.1]$ M1

Var(Y) = Var(L) + Var(T) = [0.41]   M1

Require $P(-0.1 < Y < 0.1)$ M1

$= P(Z < 0) - P(Z < -0.31...)$ or $0.5 - P(Z < -0.31)$

or $P(Z < 0.31..) - P(Z < 0)$ M1

$= 0.1217$ (tables) or $0.1226...$ (calc)   AWRT <u>0.122 – 0.123</u>   A1   5

1st M1   for a correct method for $E(Y)$, ft their $E(T)$.

2nd M1   for a correct method for Var(Y), ft their Var(T).
Must have +.

3rd M1   for dealing with the modulus and a correct probability
statement. Must be modulus free.

May be implied by e.g. $P\left(Z < \dfrac{0.2}{\sqrt{\text{their } 0.41}} \right) - 0.5$, or seeing

both 0.378... (or 0.622...) <u>and</u> 0.5

4th M1   for correct expression for the correct probability, as
printed or better. E.g. $0.5 + 0.378...$ is M0

A1   for AWRT in range.

[15]

4.   M = wt of male worker $M \sim N(78.5, 12.6^2)$

       F = wt of female worker $F \sim N(62.0, 9.8^2)$

    (a) $W = M_1 + \ldots + M_7 + F_1 + \ldots + F_8$

           $E(W) = 7 \times 78.5 + 8 \times 62.0 = 1045.50$ awrt 1050   M1 A1

           $Var(W) = 7 \times 12.6^2 + 8 \times 9.8^2 = 1879.64$ awrt 1880   M1 A1   4

    (b)   <u>Independent</u>   (used in Variance formula)   B1   1

    (c) $P(W > 1090) = P(z > \dfrac{1090 - 1045.5}{\sqrt{1879.64}})$ M1

           $= P(z > 1.03)$ awrt 1.03   A1

           $= 1 - 0.8485$ M1

           $= 0.1515$ A1   4

                    awrt(0.152)

                                                                    [9]

5.   (a) $E(R) = 20 + 10 = 30$ B1   1

       (b) $Var(R) = 4 + 0.84, = 4.84$ M1, A1   2

       (c) $R \sim N(30, 4.84)$ (Use of normal with their (a),(b))   B1ft

              $P(28.9 < R < 32.64) = P(R < 32.64) - P(R < 28.9)$ M1

              $= P\left(Z < \dfrac{32.64 - 30}{2.2}\right) - P\left(Z < \dfrac{28.9 - 30}{2.2}\right)$ stand their σ and μ   A1, A1

              $= P(Z < 1.2) - P(Z < -0.5)$ M1

              $= 0.8849 - (1 - 0.6915)$ Correct area   A1   6

              $= 0.8849 - 0.3085 = 0.5764$ (accept AWRT 0.576)

                                                                    [9]

6.   Let $W = C_1 - C_2$ <u>NB</u> $W = C_1 + C_2 \Rightarrow$ M1A0M1 only

(a) $\therefore W \sim$ N(0, 16)

Normal	M1
0; 16	A1; A1

$\therefore$ P($\mid W \mid > 6$) = 2P($W > 6$)   M1

<u>NB</u> as MR $W = C - L$ treat $= 2 \times$ P($Z > \dfrac{6-0}{\sqrt{16}}$)   M1

Standardising their σ

$\qquad\qquad\qquad\qquad\qquad = 2 \times$ P($Z > 1.5$)

Prob = 0.4346 $= 2 \times (1 - 0.9332) = \underline{0.1336}$ A1   6

(b)   Let $W = C - L$
        $\therefore W \sim$ N(5, 25)   B1; B1
                5; 25

P($W > 0$) = P($Z > \dfrac{\pm 5}{\sqrt{25}}$)   M1 A1

$\qquad = $ P($Z < 1$)   M1 ($p > 0.5$)
$\qquad = \underline{0.8413}$ A1   6

(c)   Let $W = C_1 + \ldots + C_{24} + B$
        $\therefore$ E(W) = $24 \times 360 + 100 = \underline{8500}$ B1
        var(W) = $24 \times 8 + 2^2 = \underline{196}$ B1

P($8510 \le W \le 8520$) = P$\left(\dfrac{8510 - 8500}{\sqrt{196}} \le Z \le \dfrac{8520 - 8500}{\sqrt{196}} \right)$ M1

$\qquad\qquad = $ P($0.71\ldots \le Z \le 1.43\ldots$)   awrt A1 A1ft
$\qquad\qquad = 0.9236 - 0.7611$
$\qquad\qquad = \underline{0.1625}$ A1   6
        0.161 – 0.163

(d)   <u>All</u> random variables are <u>independent</u>   B1   1

                                                                                          [19]

7.   (a)   E(D) = E(A) – 3E(B) + 4E(C)   M1
          = 20   A1
          Var(D) = Var(A) + 9Var(B) + 16Var(C)

                    Use of a²Var X   M1
                    Adding 3 Var   M1
                    ie 4 +...

          =341

          $P(D < 44) = P\left(z < \dfrac{44 - 20}{\sqrt{341}} \right)$ M1, A1ft

                    standardising their mean and sd

          = P (z < 1.30)   A1

                    awrt 1.30

          = <u>0.9032</u>   A1   9

   (b)   E(X) = 20   B1
          Var (X) = Var (A) + 3Var(B) + 16 Var (C)

                    + and 16   M1
                    3 Var (B)   M1

          = 287   A1

          $P(X > 0) = P\left(z > \dfrac{-20}{\sqrt{287}} \right)$ M1

                    standardising their mean and sd

          = P (z > –1.18)   A1

                    awrt -1.18

          = 0.8810   A1   7

                                                                                                              [16]

8.   (a)   E(X-Y) = 20 – 10 = 10   Require minus, 10   M1 A1   2

   (b)   Var(X-Y) = 5 + 4 = 9   Require plus, 9   M1 A1   2

(c)   X-Y ~ N(10,9)   Implied   B1⌡

P(13<X-Y<16) = P(X-Y<16)-P(X-Y<13)   Subtract   M1

$= P(Z< \dfrac{16-10}{3})–P(Z<\dfrac{13-10}{3})$ Standardise   M1

$= P(Z<2) – P(Z<1)$ 2&1   A1

$= 0.9772 – 0.8413 = 0.1359$ 0.1359   A1   5

[9]

1.   Part (a) was often answered well with nearly all the candidates realising they needed to form a new variable such as $J - P$ and then correctly calculating the mean and variance. Some were not sure which way round their inequality went in the resulting probability (a diagram may well have helped them) but there were a good number of correct answers seen.

The candidates' use of notation in (b) was poor with many writing $60J - 60P$ when their calculations implied they were using the correct formulation ($\Sigma J – \Sigma P$). Those who formed a correct distribution in (b) sometimes struggled to coordinate the units and they used 2 and 60 instead of 120 and 60 in their standardisation.

2.   Part (a) was answered very well with most scoring both marks. The majority secured all the marks in part (b) too but some forgot to square the 3 and the 4 and others forgot to add the variances. Most were able to write down $E(B)$ and often $Var(B)$ too but many did not clearly form a new distribution $D = B – A$ and proceed to write down the distribution of D and state the problem as $P(D > 0)$. Those who did were usually able to complete the question successfully but those who didn't often floundered being unsure how to standardize their expression.

3.   Many candidates scored well here but the usual confusion between $4S$ and T caused some to stumble and only the better candidates dealt with the modulus sign correctly in part (c).

In part (a) a number of candidates formed the distribution $X = L – 4S$ and went on to show $X \sim N(0.1, 0.89)$ but some then struggled to find $P(X > 0)$ and an answer smaller than 0.5 was often seen. In part (b) some candidates thought that $T = 4S$ but many did interpret T as $S_1 + S_2 + S_3 + S_4$ even though this was rarely explicitly stated. A common error in dealing with the probability calculation was to treat it as $2P[(L – T) < 0.1]$, others found $z = 0$ and $z = –0.31$ but then gave the answer as $0.5 + P(Z < – 0.31)$. A clear diagram, which was rarely seen, may have helped them.

4.   In part (a) a relatively large number of incorrect answers were seen for the variance, but the majority of candidates found it easy to score full marks on this question.

5.   This was a straightforward question and it was very well answered by the majority of the candidates. A minority did not understand the notation for a normal distribution and thought that N(20, 4) meant that the standard deviation was 4 not 2. The probability in part (c) was usually found correctly and errors in standardization were very rare.

6.   Many candidates did not read this question carefully and in part (a) used one cola can and one lemonade can rather than two cola cans as stated in the question. Part (b) was usually correct but in part (c) poor arithmetic and confusion of units caused many candidates to lose marks. The idea that all the random variables were independent was lost on most candidates.

7.   This question was often very well answered and a mark of 16 was not uncommon. It required no interpretation and provided the candidates recognised the need to use $a^2 \text{Var} X$ and to add variances they encountered few problems. Most candidates managed to standardize and even very weak candidates often scored 5 out of the 9 marks for part (a).

8.   This question was answered very well with a large number of candidates gaining all the marks available.